

Quantum gravity effects on compact star cores

Peng Wang¹, Haitang Yang² and Xiuming Zhang³

*Department of Applied Physics,
University of Electronic Science and Technology of China,
Chengdu, 610054, People's Republic of China*

Abstract

Using the Tolman-Oppenheimer-Volkoff equation and the equation of state of zero temperature ultra-relativistic Fermi gas based on GUP, the quantum gravitational effects on the cores of compact stars are discussed. Our results show that $2\tilde{m}(\tilde{r})/\tilde{r}$ varies with \tilde{r} . Quantum gravity plays an important role in the region $\tilde{r} \sim 10^3$. Furthermore, near the center of compact stars, we find that the metric components are $g_{tt} \sim \tilde{r}^4$ and $g_{rr} = (1 - \tilde{r}^2/6)^{-1}$. All these results are different from those obtained from classical gravity.

The configuration of a spherically symmetric static star, composed of perfect fluids, is determined by the Tolman-Oppenheimer-Volkoff (TOV) equation (in c.g.s. units) [1, 2]

$$\frac{dP}{dr} = -(\rho + P/c^2) \frac{Gm(r) + 4\pi Gr^3 P/c^2}{r[r - 2Gm(r)/c^2]}, \quad (1)$$

with

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r), \quad (2)$$

where c is the velocity of light. G is the gravitational constant. P and ρ are respectively the pressure and the macroscopic energy density measured in proper coordinates. Supplied with an equation of state and appropriate boundary conditions, eqn. (1) and eqn. (2) determine $P(r)$, $m(r)$ and $\rho(r)$. If the pressure and gravitational potential is everywhere small, i.e., $P(r) \ll \rho c^2$, $2Gm(r)/c^2 r \ll 1$, the TOV equation reduces to the fundamental equation of Newtonian astrophysics

$$\frac{dP}{dr} = -\rho(r) \frac{Gm(r)}{r^2}. \quad (3)$$

¹E-mail: pengw@uestc.edu.cn

²E-mail: hyanga@uestc.edu.cn

³E-mail: zhangxm@uestc.edu.cn

Most of the low density compact stars like white dwarfs are well described by Newtonian gravity. However, for compact stars like neutron stars and other exotic compact stars, general relativity plays an important role [3]. Although the real structures of neutron stars and other Fermi stars are very complicated, the most important physical property is given by a simple model in which the pressure of cold degenerate fermions contends against the gravitational collapse [2].

For the high density and high pressure cold Fermi stars it is worth considering the effects of quantum gravity. In the absence of a full theory of quantum gravity, effective models are useful tools to gain some features from quantum theory of gravity. One of the most important models is the generalized uncertainty principle (GUP), derived from the modified fundamental commutation relation [4, 5, 6, 7, 8, 9, 10]

$$[x, p] = i\hbar(1 + \beta p^2), \quad (4)$$

where $\beta = \beta_0 l_p^2 / \hbar^2 = \beta_0 / c^2 M_p^2$, $l_p^2 = G\hbar/c^3$, $M_p^2 = \hbar c/G$. $\hbar = h/2\pi$ is the Planck constant and β_0 is a dimensionless parameter. With this modified commutator, one can easily derive the generalized uncertainty principle (GUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2} [1 + \beta (\Delta p)^2], \quad (5)$$

which in turn gives the absolutely smallest uncertainty in positions, i.e., the minimum measurable length

$$\Delta x \geq \Delta_{\min} = \hbar \sqrt{\beta} = \sqrt{\beta_0} l_p. \quad (6)$$

Note that the model in (4) considers only the minimal uncertainty in position. In this case, the quantum mechanics structure underlying the GUP has been studied in full detail [7]. The statistics of ideal gases based on GUP has been discussed by many authors [11, 12, 13, 14, 15]. In our recent work, we have studied a system composed of zero temperature ultra-relativistic Fermi gas based on GUP [15]. The Newtonian equation with uniform pressure was employed to discuss stellar structures. In this paper, we will apply the TOV equation to investigate the cores of the stars where quantum gravity plays a leading role.

In [15], the particle number, energy density and pressure for an ultra-relativistic system were given

$$\frac{N}{V} = \frac{8\pi}{(hc)^3} E_H^3 f(\kappa), \quad (7)$$

$$\rho = \frac{8\pi}{c^2 (hc)^3} E_H^4 h(\kappa), \quad (8)$$

$$P = \frac{8\pi}{(hc)^3} E_H^4 g(\kappa), \quad (9)$$

where $E_H = c/\sqrt{\beta} = M_p c^2 / \sqrt{\beta_0}$ denotes the Hagedorn energy, introduced in [15] and $\kappa = \varepsilon_F \sqrt{\frac{\beta}{c^2}} =$

ε_F/E_H . Moreover

$$h(\kappa) \equiv \frac{1}{4} \frac{\kappa^4}{(1 + \kappa^2)^2}, \quad (10)$$

$$f(\kappa) \equiv \frac{1}{8} \left[\frac{\kappa(\kappa^2 - 1)}{(1 + \kappa^2)^2} + \tan^{-1}(\kappa) \right], \quad (11)$$

$$g(\kappa) \equiv \kappa f(\kappa) - h(\kappa). \quad (12)$$

It is worth noting that when κ increases, the pressure blows up, while the energy density and number density are both bounded. This is a manifestation of the minimal length.

Two configurations of compact stars have been addressed in [15], by applying the Newtonian limit eqn. (3) with uniform density. One is that the star is almost composed of ultra-relativistic particles. The other is that the major contribution to the mass is from non-relativistic cold nuclei. However, to discuss the core of ultra-compact stars like neutron stars, one should use TOV equations. Setting $r = r_0 \tilde{r}$, $m = m_0 \tilde{m}$, $P = P_0 \tilde{P}$ and

$$\rho = \frac{m_0}{4\pi r_0^3} \tilde{\rho} \equiv \rho_0 \tilde{\rho}, \quad P_0 = \rho_0 c^2, \quad \frac{Gm_0}{c^2 r_0} \equiv 1, \quad (13)$$

the TOV eqn.s (1) and (2) are reduced to the following dimensionless ones

$$\frac{d\tilde{P}}{d\tilde{r}} = -(\tilde{\rho} + \tilde{P}) \frac{\tilde{m} + \tilde{r}^3 \tilde{P}}{\tilde{r}(\tilde{r} - 2\tilde{m})}, \quad (14)$$

$$\frac{d\tilde{m}}{d\tilde{r}} = \tilde{r}^2 \tilde{\rho}. \quad (15)$$

When there is no introduction of quantum gravity, for a system almost composed of ultra-relativistic fermions, the equation of state is $\tilde{P} = \tilde{\rho}/3$. An exact solution is given in [16]

$$\frac{2\tilde{m}(\tilde{r})}{\tilde{r}} = \frac{3}{7}, \quad \tilde{P}(\tilde{r}) = \frac{1}{14} \tilde{r}^{-2}. \quad (16)$$

The pressure is not zero on the surface of the star. This does not meet the physical boundary conditions. However, the point is that it is an analytic solution describing the central region of compact stars with divergent pressure in the center [2]. Note that the length scale r_0 in eqn. (13) is uncertain. Thus r , m , ρ and P can be any size.

From eqn. (16), the pressure is divergent in the center. Therefore, influences from quantum gravity should be included in the discussion. Obviously, near the surface, particles are non-relativistic while in the region around the center, particles are ultra-relativistic [2]. This determines the equations of state and boundary conditions. In the vicinity of $r = 0$, the equation of state is given by eqn. (7), eqn. (8) and eqn. (9). Under the limit $\kappa \rightarrow 0$, it is straightforward to recover $P = \rho/3c^2$. Defining $r = r_0 \tilde{r}$, $m = m_0 \tilde{m}$ with

$$r_0^{-2} \equiv \frac{4\pi G}{c^4} \frac{8\pi}{(hc)^3} E_H^4, \quad (17)$$

$$m_0 \equiv 4\pi r_0^3 \frac{8\pi}{c^2 (hc)^3} E_H^4, \quad (18)$$

$$P_0 = \rho_0 c^2, \quad \rho_0 = \frac{8\pi}{c^2(hc)^3} E_H^4, \quad (19)$$

where r_0 is the minimum radius in [15]

$$r_0 = \sqrt{\frac{\pi}{4}} \beta_0 l_p \simeq \sqrt{\beta_0} \Delta_{\min}. \quad (20)$$

The expressions (19) and (20) show the system can not be arbitrary scale, determined entirely by β_0 . This indicates that our discussion is focused on the central region of compact stars. Substituting the above expressions for P and ρ (eqn. (8) and eqn. (9)) into eqn. (14) and eqn. (15), one gets

$$\frac{d\tilde{m}(\tilde{r})}{d\tilde{r}} = \tilde{r}^2 h(\kappa), \quad (21)$$

$$\frac{d\kappa(\tilde{r})}{d\tilde{r}} = \frac{-\kappa(\tilde{r}) [\tilde{m}(\tilde{r}) + \tilde{r}^3 g(\kappa)]}{\tilde{r}[\tilde{r} - 2\tilde{m}(\tilde{r})]}. \quad (22)$$

Since the density is regular in the center, one has $m(0) = 0$ as a boundary condition. After setting $\kappa_0 \equiv \kappa(0)$ as another boundary condition, eqn. (21) and eqn. (22) are integrated numerically in Table I – Table V.

In Table I to Table III, we perform the integration with different $\kappa(\tilde{r})$. Four conclusions can be drawn from these tables:

- Different from the results obtained in classical gravity, $2\tilde{m}(\tilde{r})/\tilde{r}$ varies with \tilde{r} but not a constant $3/7$. For example, with $\kappa(\tilde{r}) = 0.1$ in Table I, the deviation of $2\tilde{m}(\tilde{r})/\tilde{r}$ is about 4%.
- $2\tilde{m}(\tilde{r})/\tilde{r}$ is not sensitive to different initial value κ_0 .
- For large $\kappa(\tilde{r})$ or small \tilde{r} , quantum gravity contribution is important to the value of $2\tilde{m}(\tilde{r})/\tilde{r}$. As $\kappa(\tilde{r})$ decreases, or \tilde{r} increases, the configuration approaches the classical one obtained in [16], with a constant $2\tilde{m}(\tilde{r})/\tilde{r} = 3/7$.
- Quantum gravity plays an important role in the region $r \sim 10^3 r_0$.

Some analytic solutions can be obtained in extreme cases as follow.

- Under $\kappa \rightarrow 0$, it is easy to see that $h(\kappa) \sim \kappa^4/4$, $g(\kappa) \sim \kappa^4/12$. Then from eqn. (21) and eqn. (22), we obtain

$$\frac{2\tilde{m}(\tilde{r})}{\tilde{r}} = \frac{3}{7}, \quad \kappa(\tilde{r}) = \left(\frac{6}{7}\right)^{1/4} \tilde{r}^{-1/2}, \quad \tilde{P}(\tilde{r}) = \frac{1}{12} \kappa^4 = \frac{1}{14} \tilde{r}^{-2}, \quad \text{for large } \tilde{r}. \quad (23)$$

This solution is nothing but the classical one without quantum gravity.

- Under $r \rightarrow 0$ and $\kappa \rightarrow \infty$, eqn. (21) and eqn. (22) can be replaced by asymptotic expressions

$$\frac{d\tilde{m}(\tilde{r})}{d\tilde{r}} = \frac{1}{4} \tilde{r}^2, \quad (24)$$

$$\frac{d\kappa(\tilde{r})}{d\tilde{r}} = \frac{-\kappa(\tilde{r}) [\tilde{m}(\tilde{r}) + \tilde{r}^3 \frac{\pi}{16} \kappa(\tilde{r})]}{\tilde{r}[\tilde{r} - 2\tilde{m}(\tilde{r})]}. \quad (25)$$

The solution of these equations is

$$\tilde{m}(\tilde{r}) = \frac{\tilde{r}^3}{12}, \quad \kappa(\tilde{r}) = \frac{32}{\pi} \frac{1}{\tilde{r}^2}, \quad P(\tilde{r}) = \frac{2}{\tilde{r}^2}, \quad \text{for } \tilde{r} \rightarrow 0. \quad (26)$$

The solution (26) represents the situation where quantum gravity dominates. This happens near the center of ultra-compact stars. One can see that it is quite different from the solution of classical gravity. Table IV is the numerical result integrated for large $\kappa(\tilde{r})$, well consistent with the asymptotic solution (26).

For a spherically symmetric static compact star, the metric is given by [3]

$$g_{rr} \equiv A(r) = \left(1 - \frac{2Gm(r)}{rc^2}\right)^{-1} = \left(1 - \frac{2\tilde{m}(\tilde{r})}{\tilde{r}}\right)^{-1}. \quad (27)$$

$$g_{tt} \equiv -B(r), \quad \frac{1}{B} \frac{dB}{dr} = \frac{2G}{c^2 r^2} \left[m(r) + \frac{4\pi r^3 P}{c^2} \right] \left[1 - \frac{2Gm}{c^2 r} \right]^{-1}. \quad (28)$$

Then for $r \rightarrow 0$, from (26), we have

$$A(r) = \frac{1}{1 - \tilde{r}^2/6}, \quad B(r) \sim \tilde{r}^4. \quad (29)$$

One may compare (29) with the classical results

$$A(r) = \frac{7}{4}, \quad B(r) \sim \tilde{r}^{1/2}. \quad (30)$$

In Table V, eqn. (21) and eqn. (22) are integrated with a large initial κ_0 . It is interesting that $2\tilde{m}/\tilde{r}$ reaches a maximum value 0.734 in the vicinity of $r = 3.00 r_0$. Our calculation shows that near the center, $2\tilde{m}/\tilde{r} = \tilde{r}^2/6$ which indicates that $2\tilde{m}/\tilde{r}$ increases with \tilde{r} . On the other hand, as $\tilde{r} \rightarrow \infty$, $2\tilde{m}/\tilde{r} \rightarrow 3/7$. Therefore, the maximum of $2\tilde{m}/\tilde{r}$ at $r = 3.00 r_0$ is a turning point, where quantum gravity effect starts to dwindle. From Table V, one also finds that g_{rr} has a small range of fluctuation. A minimum $(1 - 0.279)^{-1} = 1.39$ is achieved at $r \simeq 12.5 r_0$. This minimum is about one-third of the maximum $(1 - 0.734)^{-1} = 3.76$ at $r \simeq 3.00 r_0$. We do not have good explanation for this fluctuation. It may be caused by the effectiveness of our model. Finally, g_{rr} tends to the constant 7/4 at large \tilde{r} as expected. The profile of $2\tilde{m}/\tilde{r}$ versus \tilde{r} is plotted in Fig 1. One can see that the upper limit, 8/9 on the surface of a spherically symmetric static star, is well satisfied.

In summery, we discussed the structure of ultra-compact star cores by a simple effective quantum gravity model. The model, GUP, introduces a new equation of state, determined by eqn.s (7), (8) and (9). By plugging the equation of state into TOV equations, we found some different features from previous works in literature.

Since quantum gravitational effects play an important role only in high density, we considered configurations in which a star is almost composed of ultra relativistic particles. The asymptotic

| κ_0 | $\kappa(\tilde{r})$ | $m(\tilde{r})$ | \tilde{r} | $2\tilde{m}(\tilde{r})/\tilde{r}$ |
|------------|---------------------|----------------|-------------|-----------------------------------|
| 1000.0 | 0.1 | 21.88 | 97.70 | 0.448 |
| 100.0 | | 21.87 | 97.70 | 0.448 |
| 50.0 | | 21.85 | 97.70 | 0.447 |
| 20.0 | | 21.79 | 97.60 | 0.446 |
| 10.0 | | 21.71 | 97.60 | 0.445 |
| 8.0 | | 21.65 | 97.50 | 0.444 |
| 5.0 | | 21.53 | 97.50 | 0.442 |
| 3.0 | | 21.29 | 97.40 | 0.437 |
| 1.0 | | 19.72 | 95.20 | 0.414 |
| 0.5 | | 18.36 | 87.90 | 0.418 |

Table I: Integration from κ_0 to $\kappa(\tilde{r}) = 0.1$. The value of $2\tilde{m}(\tilde{r})/\tilde{r}$ is insensitive to initial condition κ_0 . $2\tilde{m}(\tilde{r})/\tilde{r}$ has relatively large deviation from 0.429 since at $\tilde{r} = 97.7$ quantum gravity has evident effects.

| κ_0 | $\kappa(\tilde{r})$ | $\tilde{m}(\tilde{r})$ | \tilde{r} | $2\tilde{m}(\tilde{r})/\tilde{r}$ |
|------------|---------------------|------------------------|-------------|-----------------------------------|
| 1000.0 | 0.01 | 1985.47 | 9250.70 | 0.429 |
| 100.0 | | 1985.48 | 9250.80 | 0.429 |
| 50.0 | | 1985.51 | 9251.00 | 0.429 |
| 20.0 | | 1985.61 | 9251.60 | 0.429 |
| 10.0 | | 1985.76 | 9252.50 | 0.429 |
| 8.0 | | 1985.86 | 9253.00 | 0.429 |
| 5.0 | | 1986.14 | 9254.40 | 0.429 |
| 3.0 | | 1986.72 | 9257.20 | 0.429 |
| 1.0 | | 1988.16 | 9269.70 | 0.429 |
| 0.5 | | 1981.62 | 9265.40 | 0.428 |
| 0.1 | | 2016.18 | 9405.60 | 0.429 |

Table II: Integration from κ_0 to $\kappa(\tilde{r}) = 0.01$. The different initial value κ_0 has almost no effect on $2\tilde{m}(\tilde{r})/\tilde{r}$. \tilde{r} is large enough to overwhelm quantum gravity influences.

solutions near the center are given by (26) and (29). The complete picture is given by numerical calculation. Quantum gravitational effects play a leading role only in a relatively small range $\sim 10^3 r_0 = 10^3 \sqrt{\beta_0} \Delta_{\min}$. Outside this region, the solutions are determined by eqn (16) and (30). Our discussion can be applied to neutron stars, for example.

Acknowledgement

We are grateful to F. Lin for useful discussions and thank X. Guo for numerical calculations. This work is supported in part by NSFC (Grant No. 11175039 and 11005016) and Fundamental Research Funds for the Central Universities (Grant No. ZYGX2009J044 and ZYGX2009X008).

| κ_0 | $\kappa(\tilde{r})$ | $\tilde{m}(\tilde{r})$ | \tilde{r} | $2\tilde{m}(\tilde{r})/\tilde{r}$ |
|------------|---------------------|------------------------|-------------|-----------------------------------|
| 1000.0 | 0.001 | 198372.71 | 925778.50 | 0.429 |
| 100.0 | | 198372.88 | 925778.70 | 0.429 |
| 50.0 | | 198373.07 | 925778.90 | 0.429 |
| 20.0 | | 198373.65 | 925779.50 | 0.429 |
| 10.0 | | 198374.58 | 925780.30 | 0.429 |
| 5.0 | | 198376.38 | 925781.40 | 0.429 |
| 1.0 | | 198393.76 | 925802.80 | 0.429 |
| 0.1 | | 198516.29 | 925868.20 | 0.429 |
| 0.01 | | 201594.58 | 940268.10 | 0.429 |

Table III: Integration from κ_0 to $\kappa(\tilde{r}) = 0.001$. The numerical results completely match the asymptotic solution eqn. (23).

| κ_0 | $\kappa(\tilde{r})$ | $\tilde{m}(\tilde{r})$ | \tilde{r} | $2\tilde{m}(\tilde{r})/\tilde{r}$ |
|------------|---------------------|------------------------|-------------|-----------------------------------|
| 1000.0 | 20 | 2.85×10^{-2} | 0.700 | 8.15×10^{-2} |
| 500.0 | | 2.77×10^{-2} | 0.693 | 7.99×10^{-2} |
| 200.0 | | 2.52×10^{-2} | 0.672 | 7.51×10^{-2} |
| 100.0 | | 2.14×10^{-2} | 0.636 | 6.72×10^{-2} |
| 50.0 | | 1.41×10^{-2} | 0.554 | 5.10×10^{-2} |
| 30.0 | | 0.60×10^{-2} | 0.417 | 2.89×10^{-2} |

Table IV: Integration from κ_0 to $\kappa(\tilde{r}) = 20$. The large value of κ corresponds to $r \rightarrow 0$. $2\tilde{m}(\tilde{r})/\tilde{r}$ depends sensitively on \tilde{r} .

| κ_0 | $\kappa(\tilde{r})$ | $\tilde{m}(\tilde{r})$ | \tilde{r} | $2\tilde{m}(\tilde{r})/\tilde{r}$ |
|------------|---------------------|------------------------|-------------|-----------------------------------|
| 1000.0 | 100.0 | 2.32×10^{-3} | 0.303 | 1.53×10^{-2} |
| | 50.0 | 7.05×10^{-3} | 0.439 | 3.21×10^{-2} |
| | 20.0 | 2.85×10^{-2} | 0.700 | 8.14×10^{-2} |
| | 10.0 | 7.87×10^{-2} | 0.984 | 0.160 |
| | 5.0 | 0.205 | 1.365 | 0.301 |
| | 1.0 | 0.983 | 2.706 | 0.727 |
| | 0.8 | 1.100 | 3.001 | 0.734 |
| | 0.7 | 1.169 | 3.220 | 0.726 |
| | 0.5 | 1.344 | 4.052 | 0.663 |
| | 0.3 | 1.825 | 8.091 | 0.451 |
| | 0.25 | 2.359 | 12.465 | 0.279 |
| | 0.2 | 4.062 | 22.801 | 0.356 |
| | 0.1 | 21.880 | 97.689 | 0.448 |
| | 0.01 | 1985.47 | 9250.70 | 0.429 |

Table V: Integration with a fixed $\kappa_0 = 1000$. $2\tilde{m}/\tilde{r}$ reaches its maximum value 0.734 in the vicinity of $r = 3.00 r_0$. $2\tilde{m}/\tilde{r}$ has a small range of fluctuation and achieves a minimum value 0.279 at $r \simeq 12.5 r_0$. Eventually, $2\tilde{m}/\tilde{r}$ tends to the constant 7/4 at large \tilde{r} .

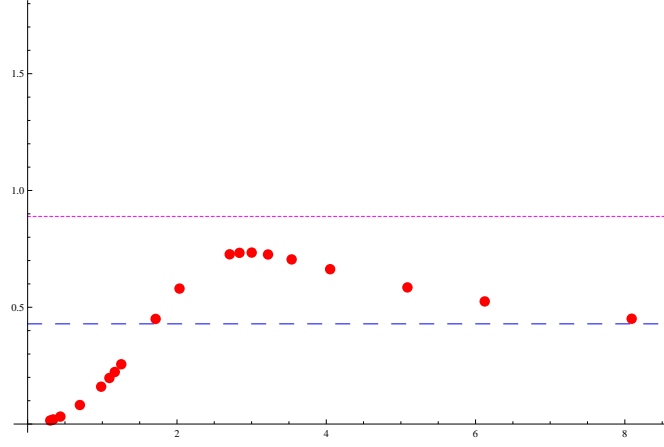


Figure 1: For a fixed $\kappa_0 = 1000$, $2m/r$ versus the radius $r = r_0 \tilde{r}$. As $\tilde{r} \rightarrow 0$, $2m/r \sim r^2$. $2m/r$ has a maximum around $\tilde{r} = 3$. $2m/r$ acquires the asymptotic value 0.429 at large r . The dashed line represents $2m/r = 0.429$ while the dotted line represents $2m/r = 8/9$, the upper limit of $2m/r$ on the surface of a spherically symmetric static star.

References

- [1] R. C. Tolman, *Static solutions of Einstein's field equations for spheres of fluid*, Phys. Rev. **55** (1939) 364.
- [2] J. R. Oppenheimer and G. M. Volkoff, *On massive neutron cores*, Phys. Rev. **55** (1939) 374.
- [3] S. Weinberg, *Gravitation and Cosmology, Chap 11*, New York: Wiley, 1972.
- [4] M. Maggiore, *A generalized uncertainty principle in quantum gravity*, Phys. Lett. **B 304** (1993) 65 [hep-th/9301067].
- [5] M. Maggiore, *The algebraic structure of the generalized uncertainty principle*, Phys. Lett. **B 319** (1993) 83 [hep-th/9309034].
- [6] L. J. Garay, *Quantum gravity and minimum length*, Int. J. Mod. Phys. **A 10** (1995) 145 [gr-qc/9403008].
- [7] A. Kempf, G. Mangano and R. B. Mann, *Hilbert space representation of the minimal length uncertainty relation*, Phys. Rev. **D 52** (1995) 1108 [hep-th/9412167].
- [8] A. Kempf, *Quantum group-symmetric fock spaces with bargmann-fock representation*, Lett. Math. Phys. **26** (1992) 1.
- [9] A. Kempf, *Quantum group Symmetric Bargmann-Fock space: integral kernels, Green functions, driving forces*, J. Math. Phys. **34** (1994) 969.
- [10] A. Kempf, *Uncertainty relation in quantum mechanics with quantum group symmetry*, J. Math. Phys. **35** (1994) 4483 [hep-th/9311147].
- [11] S. K. Rama, *Some consequences of the Generalised Uncertainty Principle: Statistical Mechanical, Cosmological and Varying Speed of Light*, Phys. Lett. **B 519** (2001) 103 [hep-th/0107255].
- [12] L. N. Chang, D. Minic, N. Okamura and T. Takeuchi, *Effect of the minimal length uncertainty relation on the density of states and the cosmological constant problem*, Phys. Rev. **D 65** (2002) 125028 [hep-th/0201017].
- [13] K. Nozari, S. H. Mehdipour, *Implications of minimal length scale on the statistical mechanics of ideal gas*, Chaos, Solitons and Fractals, **32** (2007) 1637-1644 [hep-th/0601096].
- [14] T. V. Fityo, *Statistical physics in deformed spaces with minimal length*, Phys. Lett. **A 372** (2008) 5872 [quan-th/07120891].
- [15] P. Wang, H. Yang, X. Zhang, "Quantum gravity effects on statistics and compact star configurations," JHEP **1008**, 043 (2010). [arXiv:1006.5362 [hep-th]].
- [16] C. M. Misner and H. S. Zepolsky, *High-density behavior and dynamical stability of neutron star models*, Phys. Rev. Lett. **12** (1964) 635.